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6. AUTHOR(S)
Michael V. Klibanov

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of North Carolina at Charlotte
Charlotte, NC 28223

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13. ABSTRACT (Maximum 200 words)
A number of numerical methods for multi-dimensional inverse scattering problems was developed theoretically, and some of them were tested computationally. Some related results on numerical methods for ill-posed Cauchy problems and phase retrieval problems were developed. The most promising direction of this research is an idea of the so-called "Carleman's Weight Method". This idea allows one to construct globablly convex cost functionals for a number of multi-dimensional inverse scattering problems.

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# Final Technical Report on ONR grant N00014-92-J-1008 for a Period of Time October 1, 1991 - September 30, 1995

Principal Investigator
Michael V. Klibanov
Department of Mathematics
University of North Carolina
Charlotte, NC 28223

Principal Investigator of this project has been Professor Michael V. Klibanov; co-principal investigator has been Professor Semion Gutman, Department of Mathematics, University of Oklahoma, Norman, OK 73019

Five main goals of research pursued under the funding were:

**1. Globally convergent numerical methods for multidimensional Inverse Scattering Problems (ISPs).**

While a number of numerical methods for $2-D$ ISPs has been developed in the past, convergence results were not proven. Therefore, the **most challenging** goal of the project was the development of numerical methods for $n-D$ ISPs $(n=2,3)$ with rigorously established global convergence. The first *breakthrough* result was published in [18]. It was shown in [18] how to construct globally strictly convex cost functionals for a 3-D ISP for the wave equation $u_{tt} = \Delta u + q(x)u$, with the unknown potential $q(x)$. We call this approach (CWM) *Carleman's Weight Method.*

However, a much better understanding of the advantages of CWM came recently [19, 20]. Specifically, the technique of [18] was generalized and became more convenient both for its further theoretical development and computational implementation. A most essential novelty of the recent works [19, 20], as compared with [18] consists in cutting - off the Fourier series of a function associated with the PDE solution $u(x,t)$ with respect to $t$ only, rather than with respect to both $t$ and $x$ (the number $N$ of the Fourier harmonics with respect to $t$ is a regularization parameter in this approach). This idea led us to a realization of the fact that one should employ Carleman's weights for the Laplace's operator itself, rather than for a more complicated hyperbolic/parabolic operator. The latter fact, in turn, has made CWM *computationally feasible indeed.* In addition, it should allow us to extend CWM on a number of ISPs important for applications to some ONR missions; see section 5 for some details.

Historically, for the first time, Carleman estimates were introduced into the theory of inverse problems by Bukhgeim and Klibanov in 1981 for the proofs of global uniqueness results [4]. Since 1991 Klibanov et.al. have been employing them for the proofs of convergence results for numerical methods for some linear ill-posed problems [1-3]. However, prior to [18] Carleman's weight functions have never been employed *directly* in the numerical schemes. It was our surprising discovery in [18] that Carleman's weights can lead to globally convergent methods for $n-D$ ISPs, which are essentially nonlinear.

**2. Stability theorems and numerical methods for ill-posed Cauchy problems for linear elliptic and hyperbolic PDEs [1-3].** A tight linkage between these Cauchy problems and numerical methods for ISPs was demonstrated in [18-20]. In the elliptic case, such a problem means that the Dirichlet and Neumann data are assigned on a part of the boundary only, rather than, say Dirichlet data on the whole boundary. Another option would be to solve an overdetermined problem by assigning Dirichlet data, for example on the whole boundary and Neumann data on its part (c.f.. Table 3 in [1]). In the hyperbolic case, the Dirichlet and Neumann data are given on the side of the time cylinder (or on its part), whereas data at $t=0$ are unknown.

In the sixties, French mathematicians R. Lattes and J. L. Lions suggested an elegant

and computationally feasible algorithm for these problems, the so-called quasi-reversibility method (QR) [22]. By QR, one reduces these problems to the boundary value problems for some 4th order PDEs. However, QR was not developed for the hyperbolic case, and convergence rates were not established in [22] . Likewise, QR was not applied to ISPs in [22].

An *essentially new* approach to QR was proposed in [1-3]. It consists of the employment of Carleman estimates for proofs of its convergence rate (however, Carleman's weights were not used directly in the numerical schemes, as in the case of CWM). Successful computational tests for the elliptic case were performed in [1]. Furthermore, this approach enabled us to extend QR to hyperbolic equations and to obtain Lipshitz stability estimates for this problem [2,3].

Because of the above similarities between these problems and CWM, the latter can be considered, at some extent, as a natural and far going development of our preceeding works in this direction.

**3. Computations of a 3-D ISP in a reasonable CPU time.** While 2-D ISPs have been computed by many authors, a real computation of a 3-D ISP, in a reasonable CPU time, was stated as a challenge by D. Colton and R. Kress in their book [23, p.11].

A version of the quasi-Newton method was developed and tested computationally, in the 2-D case, for an ISP for the equation $\Delta u + k^2 V(x)u = 0$ at a fixed frequency $k > 0$ [7-9,11,12]. A *breakthrough* 3-D computational result was obtained in [12] by the method of [7-9].

**4. Understanding of mathematical models for inverse problems of waves/infra-red light propagation in random media such as sea water, human tissues, etc.**

We have started from the non-stationary Scrödinger equation which describes wave propagation in random media in cases where the large scale (deterministic) features of the medium are much bigger than the typical wave length [17]. Such situations arise in ocean acoustics where waves propagate for long distances in channels characterized by small angles. A convergent numerical method for the corresponding ISP was developed in [17]. This is the first publication in which a numerical method for an $n - D$ ISP ($n > 1$) in random-media was rigorously developed.

Recently, we became focused on infra-red light propagation in turbid (i.e., diffuse) media, which leads one to ISPs either for the telegraph (i.e. hyperbolic),or for the diffusion (i.e. parabolic) equation This topic has applications in mine detection in the coastal waters and medical imaging; see section 4 for details. Preliminary computations of a corresponding ISP were performed in [13]. The goal of [13] was twofold: (i) since"diffusion" means a sort of "disorder", we have tried to check, by a simple algorithm, whether one can indeed image small inclusions hidden in a diffuse background; and (ii) to verify a new imaging idea which we call the *focusing* procedure. By this procedure, one first obtains a rough image using a rough grid. This image provides information about locations of small abnormalities, which are the main focus of the ISP solution. Then one uses a finer grid in the neighborhoods of these abnormalities (but not everywhere in the medium) to enhance the initial image. While this idea is well known for forward problems (in the Finite Element Method, for example), it was never used before for inverse problems. It was demonstrated in [13] that: (i) small

inclusions within the diffuse background can be imaged, and (ii) this focusing procedure provides a great improvement of the image quality/resolution.

**5. The phase retrieval problem.** The nature of this problem consists of recovering the phase of a signal given measurements of intensity. Note that stable phase measurements are sometimes hard to carry out in practical applications, so that often only intensity measurements are available. The phase retrieval problem can often be reduced to the recovery of the potential $V(x)$ in the Schrodinger equation $y'' + (k^2 - V(x))y = 0$ given the absolute value of the reflection coefficient $|R(k)|$ for $k \in (-\infty, \infty)$. A closely related problem consists in the determination of a function given the absolute value of its Fourier transform.

An novel theory for this problem was developed and two novel numerical methods were derived and tested computationally [5, 10, 15, 21]. We have also published a topical review in [16]. These results attracted an interest of a wide audience of mathematicians and physicists.

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